PreCalc Unit 1: Unit Circle, Graphing and Applications

Stage 1 Desired Results

ESTABLISHED GOALS:

<u>Competencies:</u>

- ***Students will demonstrate the ability to apply and extend mathematical properties in order to solve problems.***
- ***Students will demonstrate the ability to communicate and justify reasoning in order to support mathematical arguments.***

<u>Content Standards:</u>

- N.VM.1. (+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., v, |v|, ||v||, v).
- N.VM.2. (+) Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.
- N.VM.3. (+) Solve problems involving velocity and other quantities that can be represented by vectors.
- N.VM.4. (+) Add and subtract vectors.
  - N.VM.4a. Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.
  - N.VM.4b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.
  - N.VM.4c. Understand vector subtraction v – w as v + (–w), where –w is the

Transfer

*Students will be able to independently use their learning to **analyze real-life periodic phenomena.***

Meaning

ENDURING UNDERSTANDINGS

Students will understand that…

- the characteristics of trigonometric and circular functions and their representations are useful in solving real-world problems

ESSENTIAL QUESTIONS

- How are circular functions related to trigonometric functions?
- How do trigonometric and circular functions model real world problems and their solutions?

Acquisition

Students will know…

- *the values of the special right triangles and all angles on the unit circle*
- *the difference between trigonometric functions and inverse trigonometric functions*
- *the formula for arc length and its derivation*
- *the properties of the graphs of all 6 trigonometric functions*
- *the formulas and derivations of the law of sines, law of cosines and area formulas*
- *the formulas and derivations of the trigonometric identities, the reciprocal identities, the Pythagorean identities, sum, difference and double angle identities*
- *the properties of vectors and the component method of vector addition*
- *the processes of solving a trigonometric equation and formulating a trigonometric proof*
- *the domains and ranges for the trigonometric functions and inverse trigonometric functions*

Students will be skilled at…

- recognizing vector quantities as having both magnitude and direction.
- representing vector quantities by directed line segments.
- using appropriate symbols for vectors and their magnitudes (e.g., v, |v|, ||v||, v).
- finding the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.
- solving problems involving velocity and other quantities that can be represented by vectors.
- adding vectors end-to-end, component-wise, and by the parallelogram rule.
- recognizing that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.
- given two vectors in magnitude and direction form, determining the magnitude and direction of their sum.
- using sinusoidal graphs and vectors to model applications in "real life" across various disciplines.

additive inverse of w, with the same magnitude as w and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise. ● N.VM.5. (+) Multiply a vector by a scalar. ○ N.VM.5a. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as c(vx, vy) = (cvx, cvy). ○ N.VM.5b. Compute the magnitude of a scalar multiple cv using $\|\|cv\|\| = \|c\|v$. Compute the direction of cv knowing that when $\|c\|v \neq 0$, the direction of cv is either along v (for c > 0) or against v (for c < 0). ● F.TF.1. Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle. ● F.TF.2. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle. ● F.TF.3. (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosine, and tangent for $\pi - x$, $\pi + x$, and $2\pi - x$ in terms of their values for x, where x is any real number. ● F.TF.4. (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions. ● F.TF.5. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.★ ● F.TF.6. (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed. ● F.TF.7. (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the	*vocabulary:* *special right triangles, vector,, arc length*	● recognizing vector subtraction v – w as v + (–w), where –w is the additive inverse of w, with the same magnitude as w and pointing in the opposite direction. ● representing vector subtraction graphically by connecting the tips in the appropriate order. ● performing vector subtraction component-wise. ● multiplying a vector by a scalar. ● representing scalar multiplication graphically by scaling vectors and possibly reversing their direction. ● performing scalar multiplication component-wise, e.g., as c(vx, vy) = (cvx, cvy). ● computing the magnitude of a scalar multiple cv using $\|\|cv\|\| = \|c\|v$. ● computing the direction of cv knowing that when $\|c\|v \neq 0$, the direction of cv is either along v (for c > 0) or against v (for c < 0). ● recognizing the radian measure of an angle as the length of the arc on the unit circle subtended by the angle. ● explaining how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle. ● using special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$. ● using the unit circle to express the values of sine, cosine, and tangent for $\pi - x$, $\pi + x$, and $2\pi - x$ in terms of their values for x, where x is any real number. ● using the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions. ● choosing trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.★ ● using inverse functions to solve trigonometric equations that arise in modeling contexts.

<table>
<tr>
<td>solutions using technology, and interpret them in terms of the context.★
● F.TF.8. Prove the Pythagorean identity sin2(θ) + cos2(θ) = 1 and use it to find sin(θ), cos(θ), or tan(θ) given sin(θ), cos(θ), or tan(θ) and the quadrant of the angle.
● F.TF.9. (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.
● G.SRT7. Explain and use the relationship between the sine and cosine of complementary angles.
● G.SRT.8. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.★
● G.SRT.9. (+) Derive the formula A = 1/2 ab sin(C) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.
● G.SRT.10. (+) Prove the Laws of Sines and Cosines and use them to solve problems.
● G.SRT.11. (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).
● G.C5. Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.</td>
<td></td>
<td>● evaluating the solutions using technology, and interpreting them in terms of the context.★
● proving the Pythagorean identity sin2(θ) + cos2(θ) = 1 and use it to find sin(θ), cos(θ), or tan(θ) given sin(θ), cos(θ), or tan(θ) and the quadrant of the angle.
● proving the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.
● explaining and using the relationship between the sine and cosine of complementary angles.
● using trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems. (e.g. , Physics applications)
● deriving the formula A = 1/2 ab sin(C) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.
● proving the Laws of Sines and Cosines and using them to solve problems.
● applying the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).
● deriving using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality.</td>
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<tr>
<th colspan="2">Content Area Literacy Standards</th>
<th>21st Century Skills</th>
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<td colspan="2">RH.11-12.3 Evaluate various explanations for actions or events and determine which explanation best accords with textual evidence, acknowledging where the text leaves matters uncertain.
RH.11-12.4 Determine the meaning of words and phrases as they are used in a text, including analyzing how an author uses and refines the meaning of a key term over the course of a text (e.g., how Madison defines faction in Federalist No. 10).
WHST.11-12.1 Write arguments focused on discipline-specific content.
WHST.11-12.4 Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience.</td>
<td>● Solve Problems
● Communicate clearly
● Collaborate with others
● Be self-directed learners
● Reason effectively</td>
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Pre-Calc Unit 2: Sequences and Series

Stage 1 Desired Results

ESTABLISHED GOALS:

<u>*Competencies:*</u>

- ***Students will demonstrate the ability to apply and extend mathematical properties in order to solve problems.***
- ***Students will demonstrate the ability to communicate and justify reasoning in order to support mathematical arguments.***

<u>*Content Standards:*</u>

- A.SSE.4. Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems.
- F.IF3. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively by f(0) = f(1) = 1, f(n+1) = f(n) + f(n-1) for n ≥ 1.
- F.BF.1. Write a function that describes a relationship between two quantities.★
- F.BF.1a. Determine an explicit expression, a recursive process, or steps for calculation from a context.
- F.BF.2. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.★

Transfer

*Students will be able to independently use their learning to **model relationships among quantities, find a solution and evaluate the reasonableness of that solution.***

Meaning

ENDURING UNDERSTANDINGS

Students will understand that…

- sequences and series can be used to model real-life situations.
- sequences and series provide the foundation for upper level mathematics, especially calculus.
- sequences and series are a direct result of finding patterns.

ESSENTIAL QUESTIONS

- How are mathematical patterns used to simplify complex situations?
- What are the types of real-world situations where sequences and series can be used as models and prediction tools?

Acquisition

Students will know…

- *the recursive and explicit formulas for arithmetic and geometric sequences*
- *the difference between a sequence and a series (both arithmetic and geometric)*
- *the formula and derivation of an infinite series as it relates to a geometric series*
- *the notation and formula for summation notation*

<u>*vocabulary:*</u> *Sequence, Arithmetic sequence, geometric sequence, arithmetic series, geometric series, infinite series, explicit formulas, recursive formulas, convergence, divergence, summation notation, limit, end behavior.*

Students will be skilled at…

- deriving the formula for the sum of a finite geometric series (when the common ratio is not 1).
- using the formula to solve problems.
- recognizing that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.
- writing a function that describes a relationship between two quantities.
- determining an explicit expression, a recursive process, or steps for calculation from a context.
- writing arithmetic and geometric sequences both recursively and with an explicit formula.
- using them to model situations.
- writing a series using summation notation
- translating between the two forms.
- using infinite series to model problems.

		• converting between summative notation and series notation.

Content Area Literacy Standards	***21st Century Skills***
RH.11-12.3 Evaluate various explanations for actions or events and determine which explanation best accords with textual evidence, acknowledging where the text leaves matters uncertain. RH.11-12.4 Determine the meaning of words and phrases as they are used in a text, including analyzing how an author uses and refines the meaning of a key term over the course of a text (e.g., how Madison defines faction in Federalist No. 10). WHST.11-12.1 Write arguments focused on discipline-specific content. WHST.11-12.4 Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience.	• ***Solve Problems*** • ***Communicate clearly*** • ***Collaborate with others*** • ***Be self-directed learners*** • ***Reason effectively***

Pre-Calc Unit 3: Exponential and Log Functions with Apps

Stage 1 Desired Results

ESTABLISHED GOALS:

Competencies:

- ***Students will demonstrate the ability to apply and extend mathematical properties in order to solve problems.***
- ***Students will demonstrate the ability to communicate and justify reasoning in order to support mathematical arguments.***

Content Standards:

- A.SSE.3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.★
    - A.SSE.3c. Use the properties of exponents to transform expressions for exponential functions.
- F.IF.7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★
    - F.IF.7e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
- F.IF.8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
    - F.IF.8b. Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as y = (1.02)t, y = (0.97)t, y = (1.01)12t, y = (1.2)t/10, and classify them as

Transfer

*Students will be able to independently use their learning to **model relationships among quantities, find a solution and evaluate the reasonableness of that solution.***

Meaning

ENDURING UNDERSTANDINGS

Students will understand that...

- the characteristics of exponential and logarithmic functions and their representations are useful in solving real world problems.

ESSENTIAL QUESTIONS

- How do exponential functions model real world problems and their solutions?
- How do logarithmic functions model real world problems and their solutions?

Acquisition

Students will know…

- *the difference between the exponential form and the logarithmic form of an equation*
- *that a logarithm = an exponent*
- *the graphical properties and characteristics of both logarithmic and exponential equations*
- *the properties of natural logarithmic functions and "e"*
- *the inverse properties properties of exponential and logarithmic functions.*
- *the characteristics and properties of logistic equations and graphs.*
- *the characteristics and properties of growth and decay equations and graphs.*
- *the domain and range of logarithmic and exponential functions.*
- *the three rules of combining logarithmic expressions and all the properties that relate to logarithms*
- *the formula* $y = ce^{kt}$ *as it relates to the differential equation dy/dt = ky from calculus.*

Students will be skilled at…

- using the properties of exponents to transform expressions for exponential functions into logarithmic functions.
- graphing functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
- graphing exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
- using the properties of exponents to interpret expressions for exponential functions and logarithmic functions.
- solving logarithmic and exponential equations.
- solving problems involving exponential growth and decay and application of logarithms.
- using the inverse relationship between exponents and logarithms to solve problems involving logarithms and exponents.

representing exponential growth or decay. ● F.BF.1. Write a function that describes a relationship between two quantities.★ ○ F.BF.1b. Combine standard function types using arithmetic operations. ● F.BF.5. (+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents. ● F.LE.4. For exponential models,express as a logarithm the solution to abct = d where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.	*vocabulary:* *logarithm, natural logarithm, growth and decay model, "e", point of inflection, carrying capacity, logistic equation, horizontal and vertical asymptotes*	● using the formula $y = ce^{kt}$ to solve problems involving continuous growth. ● using a logistic equation to solve problems and recognizing when a problem models logistic growth. ● evaluating the logarithm using technology.

Content Area Literacy Standards	***21st Century Skills***
RH.11-12.3 Evaluate various explanations for actions or events and determine which explanation best accords with textual evidence, acknowledging where the text leaves matters uncertain. RH.11-12.4 Determine the meaning of words and phrases as they are used in a text, including analyzing how an author uses and refines the meaning of a key term over the course of a text (e.g., how Madison defines faction in Federalist No. 10). WHST.11-12.1 Write arguments focused on discipline-specific content. WHST.11-12.4 Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience.	● ***Solve Problems*** ● ***Communicate clearly*** ● ***Collaborate with others*** ● ***Be self-directed learners*** ● ***Reason effectively***

Pre-Calc Unit 4: Rational, Composite and Piecewise Functions

Stage 1 Desired Results

ESTABLISHED GOALS:

Competencies:

- ***Students will demonstrate the ability to apply and extend mathematical properties in order to solve problems.***
- ***Students will demonstrate the ability to communicate and justify reasoning in order to support mathematical arguments.***

Content Standards:

- F.IF.7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★
    - F.IF.7b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
    - F.IF.7c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
    - F.IF.7d. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.
- F.BF.1. Write a function that describes a relationship between two quantities.★
    - F.BF.1c. (+) Compose functions.
- F.BF.4. Find inverse functions.
    - F.BF.4b. (+) Verify by composition that one function is the inverse of another.

Transfer

*Students will be able to independently use their learning to **model relationships among quantities, find a solution and evaluate the reasonableness of that solution.***

Meaning

ENDURING UNDERSTANDINGS

Students will understand that…

- relations and functions can be represented in a table, numerically, graphically, algebraically and/or vertically.
- that properties of functions and function operations are used to model and analyze real-world applications and quantitative relationships.
- that rational, composite and piecewise functions are the foundation of calculus.

ESSENTIAL QUESTIONS

- Why are relations and functions represented in multiple ways?
- How are the properties of functions and functional operations useful?

Acquisition

Students will know…

- *the definition of a rational function and the properties of rational functions.*
- *the definitions of horizontal and vertical asymptotes*
- *that vertical asymptotes can be determined by analyzing the denominator of a rational function*
- *that a horizontal asymptote is equal to the limit of a function as x goes to infinity (end behavior)*
- *the definition of essential and removable discontinuities*
- *the graphs and transformations of rational functions.*
- *that composite function information can be presented in both a table and a graph.*
- *the domain and range of rational, composite and piecewise functions*

Students will be skilled at…

- graphing rational functions expressed symbolically and identifying key features of the graph, by hand in simple cases and using technology for more complicated cases.
- graphing square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
- graphing rational functions, identifying zeros when suitable factorizations are available, and showing end behavior.
- writing a function that describes a relationship between two quantities.
- composing functions.
- finding inverse functions.
- verifying by composition that one function is the inverse of another.

	• *the relationship between vertical asymptotes and end behavior.* • *the relationship between horizontal asymptotes and end behavior.* • *the meaning both algebraically and graphically of a "hole" in the graph* *vocabulary:* *composite function, piecewise functions, rational functions, limits, end behavior, continuity, discontinuity, asymptotes, hole in graph, extrema*	• identifying domain, range, x and y intercepts, extrema, endbehavior, horizontal and vertical asymptotes and holes in the graph given a rational equation

Content Area Literacy Standards	***21st Century Skills***
RH.11-12.3 Evaluate various explanations for actions or events and determine which explanation best accords with textual evidence, acknowledging where the text leaves matters uncertain. RH.11-12.4 Determine the meaning of words and phrases as they are used in a text, including analyzing how an author uses and refines the meaning of a key term over the course of a text (e.g., how Madison defines faction in Federalist No. 10). WHST.11-12.1 Write arguments focused on discipline-specific content. WHST.11-12.4 Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience.	• ***Solve Problems*** • ***Communicate clearly*** • ***Collaborate with others*** • ***Be self-directed learners*** • ***Reason effectively***

Pre-Calc Unit 5: Limits and Definition of a Derivative

Stage 1 Desired Results

ESTABLISHED GOALS:

Competencies:

- ***Students will demonstrate the ability to apply and extend mathematical properties in order to solve problems.***
- ***Students will demonstrate the ability to communicate and justify reasoning in order to support mathematical arguments.***

Content Standards:

- F.IF.4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.
- F.IF.6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

Transfer

Students will be able to independently use their learning to ***model relationships among quantities, find a solution and evaluate the reasonableness of that solution.***

Meaning

ENDURING UNDERSTANDINGS

Students will understand that…

- the concept of a limit can be used to understand the behavior of functions.
- continuity is a key property of functions that is defined using limits.
- the derivative of a function is defined as the limit of a difference quotient and can be determined using a variety of strategies.
- a function's derivative, which is itself a function, can be used to understand the behavior of the function.
- the derivative has multiple interpretations and applications including those that involve instantaneous rate of change.

ESSENTIAL QUESTIONS

- How are limits used to discover and develop important ideas, definitions, formulas and Theorems in Calculus?
- How is the derivative used to describe instantaneous rate of change and model real life situations?

Acquisition

Students will know…

- the key features of graphs of functions and their derivatives including intercepts and intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.
- the differences in the concepts of average rate of change and instantaneous rate of change both graphically and algebraically.
- the relationship of position, velocity and acceleration in terms of functions and their derivatives

Students will be skilled at…

- determining whether a function is continuous based on the calculus definition of a continuity.
- interpreting key features of graphs and tables in terms of limits and continuity.
- sketching graphs showing key features given a verbal description of the relationship.
- calculating and interpreting the average rate of change of a function (presented symbolically or as a table) over a specified interval.
- estimating the rate of change from a graph.

	• the various methods of determining limits given data presented graphically, symbolically, numerically and/or verbally. • that asymptotes and holes in the graph influence the formal definition of continuity • the squeeze theorem as it applies to limits • the technology that can be used when determining limits <u>vocabulary:</u> *limit, difference quotient, the instantaneous rate of change, the average rate of change, squeeze theorem, derivative, continuity*	• finding limits algebraically, graphically, analytically and from a table. • comparing instantaneous and average rate of change. • Identifying mathematical information from graphical, symbolic, numerical, and/or verbal representations

Content Area Literacy Standards	***21st Century Skills***
RH.11-12.3 Evaluate various explanations for actions or events and determine which explanation best accords with textual evidence, acknowledging where the text leaves matters uncertain. RH.11-12.4 Determine the meaning of words and phrases as they are used in a text, including analyzing how an author uses and refines the meaning of a key term over the course of a text (e.g., how Madison defines faction in Federalist No. 10). WHST.11-12.1 Write arguments focused on discipline-specific content. WHST.11-12.4 Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience.	• ***Solve Problems*** • ***Communicate clearly*** • ***Collaborate with others*** • ***Be self-directed learners*** • ***Reason effectively***